

Giant Gate-controlled Room Temperature Odd-parity Magnetoresistance in Magnetized Bilayer Graphene

Divya Sahani¹, Sunit Das², Kenji Watanabe³, Takashi Taniguchi⁴, Amit Agarwal^{2*} and Aveek Bid^{1†}

¹*Department of Physics, Indian Institute of Science, Bangalore 560012, India*

²*Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India*

³*Research Center for Functional Materials, National Institute for Materials Science, 1-1 Namiki, Tsukuba 305-0044, Japan*

⁴*International Center for Materials Nanoarchitectonics,
National Institute for Materials Science, 1-1 Namiki, Tsukuba 305-0044, Japan*

In this Letter, we report the discovery of giant room-temperature odd-parity magnetoresistance (OMR) in a bilayer graphene (BLG) heterostructure interfaced with $\text{Cr}_2\text{Ge}_2\text{Te}_6$ (CGT). We show that the OMR is electrostatic gate-voltage tunable, exhibiting maximum value near the graphene band edges and diminishing rapidly with increasing charge carrier density. Our theoretical analysis reveals that the OMR originates from the coupling of out-of-plane components of Berry curvature and orbital magnetic moment with the applied magnetic field, signaling intrinsic time-reversal symmetry (TRS) breaking in the system. This TRS breaking persists at room temperature within the CGT/BLG heterostructure despite the bulk CGT exhibiting ferromagnetic transition at 65 K. Our study opens new avenues for probing band geometric phenomena through magnetotransport measurements, especially in quantum materials where the anomalous Hall effects are immeasurably small due to the extrinsic scattering-dependent contributions canceling Berry curvature contributions.

Introduction:– Magnetotransport measurements are essential tools for probing the symmetry and band topology of materials [1]. Such studies have led to remarkable technological breakthroughs; examples include giant magnetoresistance and tunneling magnetoresistance for advanced storage and memory devices [2–4]. Typically, in high magnetic fields, materials exhibit oscillating magnetoresistance (Shubnikov-de Haas effect), which provides insights into their band properties [5–7]. At low magnetic fields, the longitudinal magnetoresistance $R_{ab}(B)$ in non-magnetic materials follows Onsager’s reciprocity relation, $R_{ab}(B) = R_{ba}(-B)$. This results in symmetric longitudinal and mixed (symmetric/antisymmetric) off-diagonal components of resistivity [8, 9].

However, this conventional understanding fails in systems that intrinsically break time-reversal symmetry (TRS), where an antisymmetric, linear- B longitudinal magnetoresistance emerges [10–26]. This odd-parity magnetoresistance (OMR) is a definitive signature of the intrinsic TRS breaking in quantum materials, and it offers a critical probe of electronic band topology. The TRS breaking in magnetic systems is also probed by the Berry curvature-driven anomalous Hall effect (AHE) [27]. However, extrinsic scattering contributions to the AHE can sometimes negate the Berry curvature contribution and make the AHE signal vanishingly small [28–30].

In this Letter, we report a giant OMR in bilayer graphene (BLG)/ $\text{Cr}_2\text{Ge}_2\text{Te}_6$ (CGT) heterostructure, reaching up to 40% at room temperature – the highest reported to date [23]. We demonstrate that this OMR is gate-tunable, peaks near the band edges, and diminishes with increasing charge carrier density. Remarkably, no significant hysteresis in magnetic measurements or AHE is observed, and the sign of the OMR reverses between the device edges [25]. Our analysis shows

that this OMR arises from the out-of-plane Berry curvature and orbital magnetic moment in the TRS-broken BLG/CGT system, enabled by proximity-induced magnetic exchange coupling [31–33]. Our findings position OMR as a powerful tool for detecting TRS breaking in quantum materials, particularly in cases where the anomalous Hall signal is too small to be detected.

We fabricated the heterostructures of CGT/BLG/hBN on $\text{Si}^{++}/\text{SiO}_2$ substrates using the dry transfer technique [34]; see Supplemental Material (SM) Section S1 for fabrication details. In this Letter, we present the data from a device labeled **D1**. We compare the data with those from a heterostructure of hBN/BLG/hBN (labeled **D2**). Data for other CGT/BLG/hBN devices are provided in SM Sections S6 and S7. Electrical transport measurements were conducted using a low-frequency lock-in detection technique at a variable temperature cryostat at a bias current 10 nA. CGT is an electrical insulator at low temperatures [35]. Consequently, electrical transport only takes place through the bilayer graphene channel. The dual-gated geometry allows independent control of the charge carrier density n and the vertical displacement field D .

Charge transfer between BLG and CGT:– The variation of the longitudinal resistance with gate voltage shows that the charge neutrality point of the BLG is shifted to a positive gate voltage (Sec. S2 of SM), indicating that the BLG is hole-doped as a result of charge transfer occurring at the interface with CGT [36–40]. Hall resistance measurement confirms that the hole-doping is $n_0 = -9.6 \times 10^{15} \text{ m}^{-2}$ (Fig. S2(b) of SM). Such large charge transfer between CGT and graphene has been noted in previous studies [36, 40]. The charge transfer magnitude increases with increasing CGT thickness; see Fig. S2(c) and Table S2 of SM. Contrast this with the

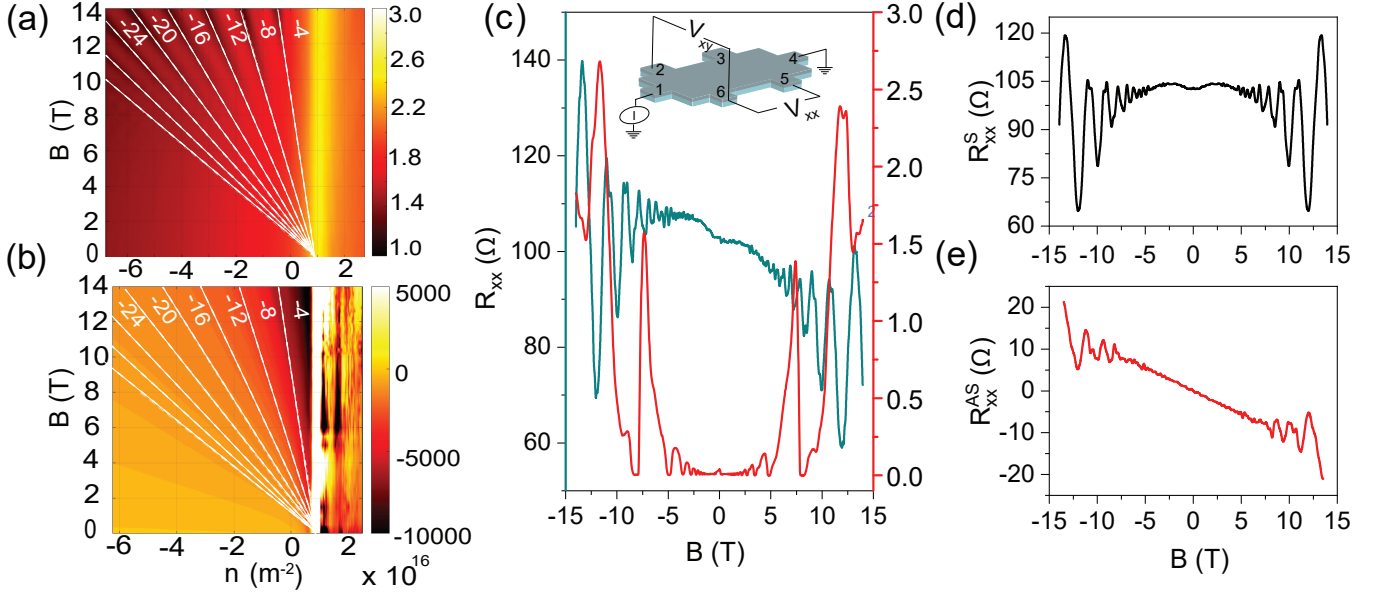


FIG. 1. **Device characteristics and antisymmetric magnetoresistance:** (a) Contour plot of longitudinal magnetoresistance R_{xx} in $n - B$ plane. (b) Contour plot of transverse magnetoresistance R_{xy} in $n - B$ plane. The white dashed lines in (a) and (b) indicate the characteristic quantum Hall states of bilayer graphene that appear at Landau level filling factors $\nu = 4N$ with $N \in \mathbb{I}$. (c) Comparison of the longitudinal magnetoresistance $R_{14,65} = (V_{65}/I_{14})$ for a hBN/BLG/hBN heterostructure (red line; right-axis) and a hBN/BLG/CGT heterostructure (teal line, left-axis). The data were measured at 2 K. Inset: Schematic of the device showing the contact numbers and measurement configurations. (d) Plot of the symmetric component of the magnetoresistance $R_{14,65}^S = [R_{14,65}(B) + R_{14,65}(-B)]/2$ versus B for the hBN/BLG/CGT device. (e) Plot of anti-symmetric component of the magnetoresistance $R_{14,65}^{AS} = [R_{14,65}(B) - R_{14,65}(-B)]/2$ versus B for the hBN/BLG/CGT device.

R - V_g plot of device **D2**—the charge neutrality point is at $V_g = 0.04$ V indicating negligible $n_0 = -1.53 \times 10^{14} \text{ m}^{-2}$ (Sec. S2 of SM).

Fig. 1(a) and Fig. 1(b) are plots of the longitudinal resistance R_{xx} and the transverse resistance R_{xy} , as a function of the carrier density n and magnetic field B . The white dashed lines mark the signature quantum Hall states of bilayer graphene occurring in multiples of four, establishing that the electrical transport in this heterostructure occurs through the BLG channel. The Landau fan diagram starts from $n \sim 8 \times 10^{15} \text{ m}^{-2}$, establishing again the hole-doping of the BLG channel by CGT.

Observation of room temperature OMR:— In Fig. 1(c), the solid teal line represents the plot of the longitudinal magnetoresistance $R_{14,65}(B)$ as a function of the magnetic field B measured at $n = -6.28 \times 10^{16} \text{ m}^{-2}$. The notation $R_{ij,kl}(B)$ indicates that the current is driven between the probes i and j in the presence of a perpendicular magnetic field B . At the same time, the potential difference is measured between the probes k and l (see inset of Fig. 1(c)). Surprisingly, the magnetoresistance has a significant asymmetry in the B -field. In contrast, the longitudinal magnetoresistance measured for the pristine bilayer graphene encapsulated between hBN layers, **D2**,

is symmetric in the B -field (Fig. 1(c) right-axis; solid red line).

Figure 1(d) is a plot of the symmetric (even-parity) component of longitudinal magnetoresistance, defined as $R_{14,65}^S = [R_{14,65}(B) + R_{14,65}(-B)]/2$. The antisymmetric component $R_{14,65}^{AS} = [R_{14,65}(B) - R_{14,65}(-B)]/2$, which we call OMR, is presented in Fig. 1(e). We note two essential characteristics of the OMR. Firstly, its change with the magnetic field is enormous, approaching 40% at 14 T (Fig. S3(a) of SM). This value is the highest reported to date [14, 25], see SM Table S1. Secondly, the OMR is linearly dependent on the B field (onto which quantum oscillations are superimposed). Note that we do not observe any hysteresis in the magnetoresistance of the BLG/CGT devices down to 2 K (see Sec. S3 and Fig. S3(c) of the SM for data).

We observe a strong electrostatic gate control on the magnitude of OMR. In Fig. 2(a), $R_{14,65}^{AS}$ is plotted for three different charge carrier densities $n = -6 \times 10^{16} \text{ m}^{-2}$ (green curve), $n = -2 \times 10^{16} \text{ m}^{-2}$ (purple curve) and $n = -1 \times 10^{16} \text{ m}^{-2}$ (red curve). One can see that the OMR depends strongly on the density of hole doping of the BLG channel. We quantify the magnitude of the OMR using the parameter $\alpha = dR_{ij,kl}^{AS}/dB$ and find that the magnitude of α decreases rapidly with

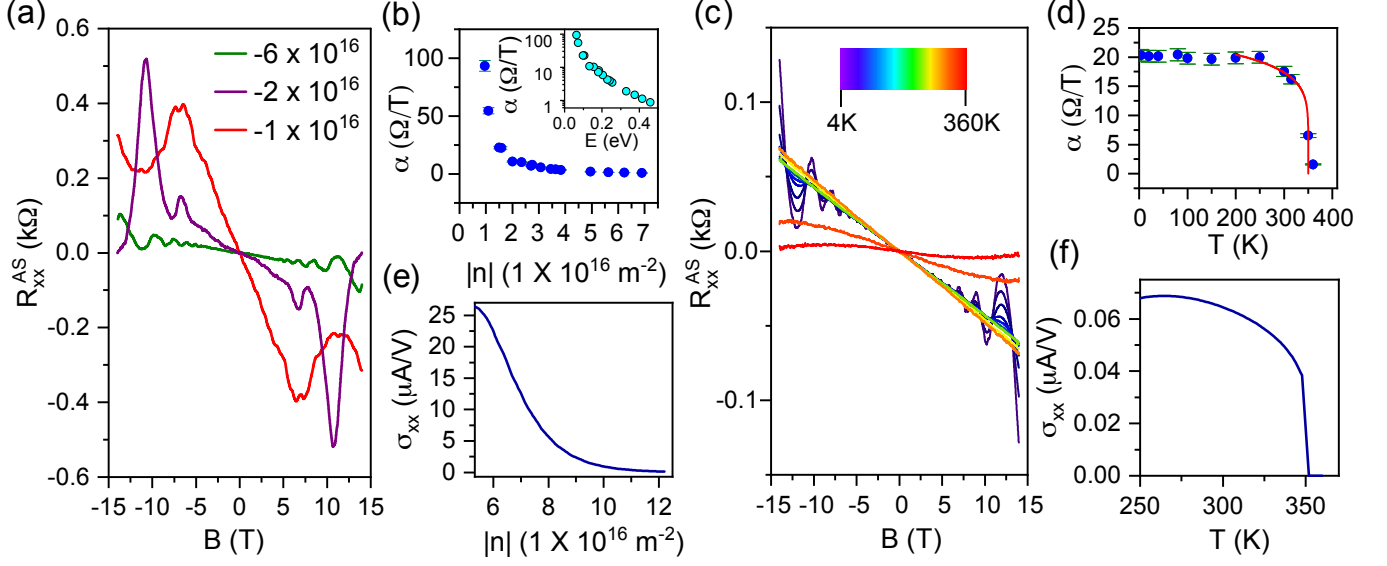


FIG. 2. **Number density and temperature dependence of OMR:** (a) Plots of the $R_{14,65}^{AS}$ versus B at three representative number densities; $n = -6 \times 10^{16} \text{ m}^{-2}$ (green curve), $n = -2 \times 10^{16} \text{ m}^{-2}$ (purple curve), and $n = -1 \times 10^{16} \text{ m}^{-2}$ (red curve). The measurements were done at $T = 2 \text{ K}$. (b) Plot of $\alpha = (dR_{xx}^{AS}/dB)$ versus n ; the measurement was performed at $B = 5 \text{ T}$ and $T = 2 \text{ K}$. Inset: Plot of $\alpha = (dR_{xx}^{AS}/dB)$ versus energy E . We consider $E \approx (\hbar^2 k^2)/(2m)$ where $k \approx \sqrt{2\pi n}$. n is extracted from Hall measurements. Effective mass of BLG is $m \sim 0.034m_e$ [41], with m_e being the electronic mass. (c) Plots of $R_{14,65}^{AS}$ versus B at a few representative temperatures between the $T = 2 \text{ K}$ and $T = 360 \text{ K}$. The data were taken at $n = -2 \times 10^{16} \text{ m}^{-2}$. (d) Plot of α versus T measured at $B = 5 \text{ T}$ and $n = -2 \times 10^{16} \text{ m}^{-2}$ (blue filled symbols). The red line shows the fit to $\alpha = \alpha_0(1 - T_c/T)^\beta$. (e) Calculated antisymmetric component of the magnetoconductivity versus n considering $B = 1 \text{ T}$ and $\tau = 1$ picosecond. (f) Calculated antisymmetric component of the magnetoconductivity versus T considering $B = 1 \text{ T}$ and $\tau = 1$ picosecond.

increasing $|n|$ (Fig. 2(b)). The magnitude of α decreases significantly as the energy goes from 50 meV to 100 meV, with α reducing to 1/10th of its maximum value. (inset of Fig. 2(b)). This observation establishes that the OMR is maximum at the band edges, providing the first clue about its origin.

Fig. 2(c) shows the plot of $R_{14,65}^{AS}$ versus B at different temperatures ranging between 2 – 360 K measured at a hole number density $|n| = 2 \times 10^{16} \text{ m}^{-2}$. As expected, the SdH oscillations disappear at higher temperatures. Surprisingly, the amplitude of $R_{14,65}^{AS}$ remains almost unchanged till about $T = 250 \text{ K}$ before beginning to fall precipitously, finally disappearing at around $T \sim 360 \text{ K}$ (see Fig. 2(d)). The observation of a gate-tunable giant OMR, which persists till room temperature and beyond, is the central result of this Letter.

Before we comprehensively explain the origin of the OMR, we rule out two artifacts that can lead to an OMR. A trivial origin of OMR in semiconductors is inhomogeneities in the device where a part of the transverse Hall voltage is reflected along the longitudinal direction of current flow [42–44]. In SM Sec. S4, we rule out this interpretation. Another possible origin can be intermixing from the Hall signal. We exclude this possibility by noting that α has the same sign for electron and hole-

doping of BLG while R_{xy} changes its sign between these two regimes (see Sec. S5 of SM).

Berry curvature induced OMR:– The OMR in TRS broken systems can be understood from a semi-classical transport framework, including the co-existence of topological bands and magnetic order. Using the Boltzmann transport formalism, we show that the linear-in- B longitudinal conductivity arises from the orbital magnetic moment and band geometric quantities such as the Berry curvature. Assuming the source-drain bias to be in the x -direction, we obtain the linear longitudinal magnetoconductivity, $\sigma_{xx}^{AS}(B)$, to be (see Sec. S8 of SM for details)

$$\sigma_{xx}^{AS} = -2e^2\tau \int_{p,\mathbf{k}} v_x^m v_x^0 \partial_{E_0} f_p^0 + \frac{e^3\tau}{\hbar} \int_{p,\mathbf{k}} (\boldsymbol{\Omega} \cdot \mathbf{B}) v_x^0 v_x^0 \partial_{E_0} f_p^0 + e^2\tau \int_{p,\mathbf{k}} v_x^0 v_x^0 [\partial_{E_0}(\mathbf{m} \cdot \mathbf{B}) \partial_{E_0} f_p^0 + (\mathbf{m} \cdot \mathbf{B}) \partial_{E_0}^2 f_p^0] \quad (1)$$

For brevity, we denote $\int_{p,\mathbf{k}} \equiv \sum_p \int \frac{d^2k}{(2\pi)^2}$ and we do not explicitly mention the band index p in the physical quantities. In Eq. (1), $\mathbf{v}^0 = \nabla_{\mathbf{k}} E_0/\hbar$ is band velocity, and $\mathbf{v}^m = \nabla_{\mathbf{k}}(-\mathbf{m} \cdot \mathbf{B})/\hbar$ is the correction to band velocity arising from the coupling of the orbital magnetic moment $\mathbf{m}$ to the magnetic field. τ is the scattering time,

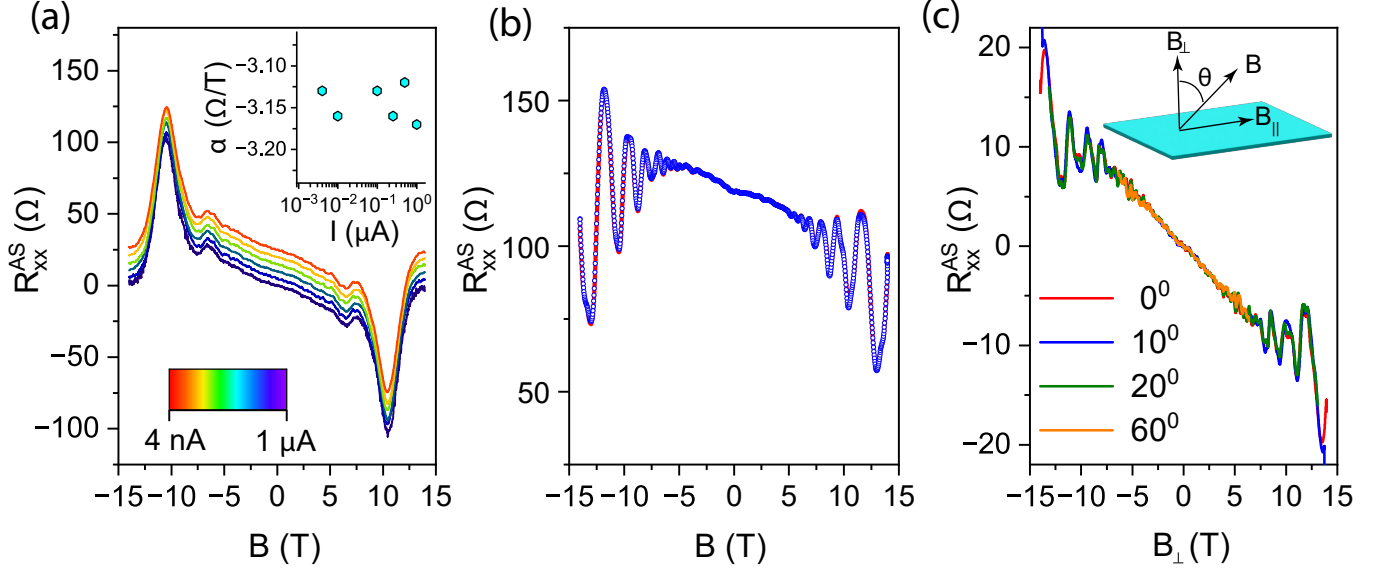


FIG. 3. **Current and parallel magnetic field dependence of antisymmetric magnetoresistance:** (a) Plots of $R_{14,65}^{AS}$ for different bias currents ranging from 4 nA – 1 μ A. Each plot has been vertically offset by 5 Ω for clarity. Inset: Plot of α versus I showing the independence of the OMR on the magnitude of I . (b) Plots of $R_{14,65}^{AS}$ (solid red line) and $R_{41,56}^{AS}$ (open blue symbols). (c) Plots of $R_{12,34}^{AS}$ versus $B_{\perp}$ for different values of the angle θ between the magnetic field and the perpendicular to the plane of the device (see inset of the figure). All the measurements were done at 2 K.

and f_p^0 is the equilibrium Fermi-Dirac distribution function. Interestingly, Ω , $\mathbf{m}$ and $\mathbf{v}^0$ ($\mathbf{v}^m$) are odd (even) functions of $\mathbf{k}$ in presence of TRS. Consequently, all the integrals in Eq. (1) become an odd function of $\mathbf{k}$ and vanish in systems preserving TRS, consistent with Onsager's reciprocity relation. Hence, the Berry curvature and orbital magnetic moment-induced linear- B longitudinal magnetoconductivity can only be finite in a TRS-broken system. Several other theoretical models have been proposed to explain OMR. However, none of these frameworks can account for the OMR observed in our samples (see the Appendix for details).

In Fig. 2(e), we present the theoretically calculated variation of the antisymmetric magneto-conductivity ($\sigma_{xx}^{AS} \propto \rho_{xx}^{AS} \propto B$) with the hole density $|n|$ for the model Hamiltonian (see Sec. S8 of SM) of bilayer graphene proximitized with a single layer of CGT [45]. We observe that σ_{xx}^{AS} (and, consequently, the antisymmetric part of the resistivity, $\rho_{xx}^{AS} \approx \sigma_{xx}^{AS} \rho_{xx}^S / \sigma_{xx}^S$) decreases with increasing number density. Here, ρ_{xx}^S and σ_{xx}^S are the symmetric part of the longitudinal resistivity and the conductivity, respectively (see SM Sec. S8 for more details). The above analysis qualitatively captures our experimental finding of α decreasing on increasing $|n|$, shown in Fig. 2(b).

Signature of magnetic phase transition in OMR:– The T -dependence of OMR hints at a large enhancement of the magnetic ordering temperature in bilayer CGT on bilayer graphene heterostructure. This is similar to the dramatic Curie temperature enhancement observed in

ultra-thin 2D materials due to substrate effect [46–50]. To explain the observed T -dependence of α , we make an implicit assumption that the OMR is proportional to the magnetization in the system [51]. As T approaches the critical temperature T_c , the magnetization of the system should vanish as $M = M_0(1 - T_c/T)^\beta$ (β being the critical exponent quantifying the dependence of M on T). The intrinsic time-reversal symmetry of the system is restored for $T > T_c$ in the paramagnetic phase where $M = 0$. Consequently, the antisymmetric part of the magnetoconductivity should become zero beyond the critical temperature. In the spirit of the above argument, we fit the data in Fig. 2(d) with the relation $\alpha = \alpha_0(1 - T_c/T)^\beta$; the fits yield $T_c \sim 350$ K and $\beta \sim 0.198$. This value of β matches very well with that previously extracted from magnetization measurements in CGT; $\beta = 0.240 \pm 0.006$ [52]. We show a plot of the calculated antisymmetric magnetoconductivity σ_{xx}^{AS} versus T in Fig. 2(f) using these experimentally obtained values of T_c and β for the proximity induced magnetization. The resulting dependence of calculated σ_{xx}^{AS} on T captures the experimental data exceptionally well.

Fig. 3(a) is the plot of $R_{14,65}^{AS}$ for different values of bias current I measured at 2 K; the plots have been vertically offset by 5 Ω for clarity. The magnitude of OMR is independent of the strength of the driving current varied over several orders of magnitude, as is apparent from the plot of α versus I (inset of Fig. 3(a)). Furthermore, Fig. 3(b) compares $R_{14,65}^{AS}$ (the current is driven from con-

tact 1 with contact 4 grounded) and $R_{41,56}^{AS}$ (the current is driven from contact 4 with contact 1 grounded); these two plots overlap perfectly. These observations establish the OMR as a reciprocal, linear response transport phenomenon with an origin distinct from recent reports where the OMR had a strong current dependence [25].

In 2D materials, only the out-of-plane component of the orbital magnetic moment (m_z) and the Berry curvature (Ω_z) are finite. The m_z couples to the out-of-plane magnetic field and modifies the band energy via Zeeman-like coupling: $E \rightarrow (E_0 - \mathbf{m} \cdot \mathbf{B})$. Similarly, a term $\propto \Omega \cdot \mathbf{B}$ modifies the phase space volume element (see Eq. (1) and Sec. S8 of SM). Thus, the in-plane component of the magnetic field should play no role in the observed antisymmetric longitudinal conductivity (or resistivity) induced by the m_z and Ω_z . We verified this assertion by measuring OMR in a tilted magnetic field. For these measurements, the device was rotated with respect to the magnet's axis. One can see from Fig. 3(c) that for a given value of the perpendicular component of the magnetic field, $B_\perp$, the OMR is independent of the angle between the magnetic field and the perpendicular to the plane of the device, θ (and consequently of the parallel component of the magnetic field, $B_\parallel$). This independence of OMR on $B_\parallel$ further validates the consistency of our theoretical understanding with the experimental observation. It also rules out other possible spin-based mechanisms [25, 53] of generating OMR via the spin-Zeeman coupling (see Sec. S9 of SM for more details).

In Fig. 4(a), we plot the OMR measured for the bottom edge (red curve, $R_{14,65}^{AS}$) and the top edge (black curve, $R_{14,23}^{AS}$) of the device. The OMR switches sign between the two edges. Given that the direction of the current and the magnetic field in these two measurements remain identical, the flipping of the sign of OMR between the two edges indicates the presence of trivial 1-D edge modes in addition to the 2-D bulk mode [25]. Such trivial non-topological edges have been observed in etched graphene devices and attributed to charge accumulation at the edges [54]. We do not completely understand the origin of these trivial edge states in our CGT/BLG devices. On the other hand, as detailed in Ref. [25], the 2-probe longitudinal resistance $R_{14,14}$ must be B -field symmetric; Fig. 4(b) establishes that this indeed is the case.

Conclusion:– Bilayer graphene (BLG) in proximity to a magnetic insulator, such as $\text{Cr}_2\text{Ge}_2\text{Te}_6$, can exhibit the anomalous Hall effect and exchange splitting during field cooling experiments. However, the anomalous Hall signal can become difficult to detect in ultra-thin ferromagnetic layers due to possible cancellation between Berry curvature and extrinsic scattering-dependent contributions. We demonstrate that odd-parity magnetoresistance (OMR) provides a robust alternative for detecting broken time-reversal symmetry in these systems. Our observation of OMR in BLG/CGT heterostructures at

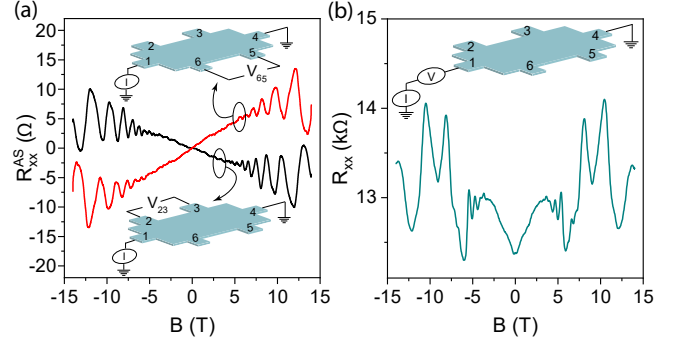


FIG. 4. Edge property of antisymmetric magnetoresistance: (a) OMR measured at the two opposite edges of the channel; $R_{14,23}^{AS}$ (black curve) and $R_{14,65}^{AS}$ (red curve). (b) Plot of the 2-probe longitudinal magnetoresistance $R_{14,14}$ versus B . The 2-probe resistance does not have any antisymmetric component. The measurements were done at $T = 2$ K, $\theta = 0^\circ$, and $n = -1 \times 10^{16} \text{ m}^{-2}$.

room temperature is significant, and it arises from the interplay between band geometric properties and magnetic exchange interaction induced by CGT.

Crucially, our findings suggest that magnetic ordering in the BLG/CGT heterostructure persists up to room temperature despite the ferromagnetic Curie temperature (T_C) of bulk CGT being around 65 K. The substantial enhancement of T_C arising from strain and substrate effects has been predicted in a variety of materials [49, 50, 55–61]. This highlights a promising approach for increasing T_C in low-dimensional ferromagnets. Our discovery motivates further explorations of the magnetic and topological properties of the BLG/2D ferromagnet heterostructures, unlocking new avenues for next-generation magnetic sensing and memory devices.

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END MATTER

Appendix: Possible mechanisms of OMR— In this Appendix, we discuss some of the existing theories of linear- B magnetoresistance and explain why they do not account for our experimental observations.

Parish-Littlewood model: The Parish-Littlewood model explains the OMR in a large inhomogeneous system [62]. In the presence of significant inhomogeneity in the system, the current flow gets distorted, and a part of the transverse Hall current appears in the longitudinal direction, which is linear in a magnetic field. This current flow pattern gives rise to the linear MR. However, our analysis shows no significant inhomogeneity, which can cause linear MR. The carrier density inhomogeneity can also cause the linear B MR, observed in semiconductors [42]. As discussed in Sections S4 and S5 of the SM, the inhomogeneity pictures can not explain OMR in our devices.

Abrikosovs quantum theory: Abrikosov's quantum theory predicts linear-in- B MR for the systems in the extreme quantum limits [63, 64]. Only the lowest Landau level should be filled in the extreme quantum limit, requiring very low carrier density and temperature. Our experimental observation of room temperature OMR down to a very low magnetic field regime rules out Abrikosov's quantum theory as a possible explanation for antisymmetric MR.

Guiding center motion: In Ref. [65], Song *et al.* proposed a semiclassical mechanism for linear- B -MR, where the guiding center motion dominates charge transport. Specifically, when the disorder potential varies on a scale that is large compared to the cyclotron radius, the Hall angle (σ_{xy}/σ_{xx}) can become independent of the magnetic field. The longitudinal MR can be written as $\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{\mathcal{G}}{\sigma_{xy}}$ with $\mathcal{G} = \frac{\tan \theta_H}{1 + \tan^2 \theta_H}$, $\tan \theta_H = \sigma_{xy}/\sigma_{xx}$. Consequently, when $\tan \theta_H$ is independent of B , and using the relation $\sigma_{xy} = ne/B$, we can see that the ρ_{xx} becomes linear in B . As explained in Ref. [65], the theory only works for three-dimensional materials. Hence, this theory does not concern our experimental system and can not explain the observation of OMR.

Hotspot scattering in charge density wave phases: Another theory for linear MR is the 'hotspot' theory in the charge density wave phase of materials [66–68]. The electron scattering around the 'hotspot' of the Fermi surface in the presence of CDW fluctuations leads to the linear field dependence of the scattering time. The field-dependent scattering finally gives rise to the linear- B -MR. However, no experimental evidence of a charge density wave phase in the graphene/CGT system has been found. Therefore, the 'hotspot' theory of electron scattering is not applicable to our experimental results.

Linear band structures and spin-Zeeman coupling: In Ref. [53], a phenomenological model for linear MR was

proposed based on linear band structure. It was argued that for linear bands, the change in spin-Zeeman energy modified density of states (at the Fermi level) between two spin channels with the magnetic field is linearly proportional to the applied magnetic field. This gives rise to the $MR \propto B$ in systems where electron-phonon scattering dominates (scattering time $\propto \text{DOS}^{-1}$). In the bilayer graphene system, the low-energy bands are approximately quadratic in the crystal momentum. Thus, Zeeman splitting modified linear band structure theory for linear- B -MR fails to capture our experimental observations.

In Ref. [25], the OMR in the 1D edge channel of InAs/(Ga, FeSb) heterostructure was explained to arise from the spin-Zeeman coupling. The magnetic field modifies the band energy via the spin-Zeeman coupling term. Asymmetric scattering between two helical spin-orbit coupled bands makes the Drude conductivity linearly magnetic field dependent. When the OMR is associated with the spin-Zeeman term, we expect it to be sensitive to the in-plane and out-of-plane magnetic field components. However, as mentioned earlier, the OMR in BLG/CGT is insensitive to the in-plane magnetic field component, which rules out the theory of OMR presented in Ref. [25] for our system.

These analyses rule out all aforementioned mechanisms as explanations for the observed OMR in our BLG/CGT devices. Instead, our results are best explained by the band geometry induced linear- B magneto-conductivity in the presence of time-reversal symmetry breaking due to the magnetic proximity effects, as detailed in the main text and SM.

* amitag@iitk.ac.in

† aveek@iisc.ac.in

- [1] H.-Z. Lu and S.-Q. Shen, Quantum transport in topological semimetals under magnetic fields, *Frontiers of Physics* **12**, 127201 (2017).
- [2] M. N. Baibich, J. M. Broto, A. Fert, F. N. Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, Giant magnetoresistance of (001)Fe/(001)Cr magnetic superlattices, *Phys. Rev. Lett.* **61**, 2472 (1988).
- [3] S. Yuasa, T. Nagahama, A. Fukushima, Y. Suzuki, and K. Ando, Giant room-temperature magnetoresistance in single-crystal Fe–MgO–Fe magnetic tunnel junctions, *Nature Materials* **3**, 868 (2004).
- [4] R. Katti, Giant magnetoresistive random-access memories based on current-in-plane devices, *Proceedings of the IEEE* **91**, 687 (2003).
- [5] A. Alexandradinata and L. Glazman, Fermiology of topological metals, *Annual Review of Condensed Matter Physics* **14**, 261 (2023).
- [6] P. Tiwari, M. K. Jat, A. Udupa, D. S. Narang, K. Watanabe, T. Taniguchi, D. Sen, and A. Bid, Experimental observation of spin-split energy dispersion in high-mobility single-layer graphene/wse2 heterostructures, *npj 2D Ma-*

- terials and Applications **6**, 68 (2022).
- [7] S. Das, S. Chakraverty, and A. Agarwal, **Nonlinear landau fan diagram and aperiodic magnetic oscillations in three-dimensional systems** (2024), [arXiv:2403.03765](#).
 - [8] L. Onsager, Reciprocal relations in irreversible processes. i., **Physical review** **37**, 405 (1931).
 - [9] P. Mazur and S. de Groot, On onsager's relations in a magnetic field, **Physica** **19**, 961 (1953).
 - [10] A. Cortijo, Linear magnetochiral effect in weyl semimetals, **Phys. Rev. B** **94**, 241105 (2016).
 - [11] G. Sharma, P. Goswami, and S. Tewari, Chiral anomaly and longitudinal magnetotransport in type-ii weyl semimetals, **Phys. Rev. B** **96**, 045112 (2017).
 - [12] K. Das and A. Agarwal, Linear magnetochiral transport in tilted type-i and type-ii weyl semimetals, **Phys. Rev. B** **99**, 085405 (2019).
 - [13] A. Knoll, C. Timm, and T. Meng, Negative longitudinal magnetoconductance at weak fields in weyl semimetals, **Phys. Rev. B** **101**, 201402 (2020).
 - [14] C. Xiao, H. Chen, Y. Gao, D. Xiao, A. H. MacDonald, and Q. Niu, Linear magnetoresistance induced by intrascattering semiclassics of bloch electrons, **Phys. Rev. B** **101**, 201410 (2020).
 - [15] V. A. Zyuzin, Linear magnetoconductivity in magnetic metals, **Phys. Rev. B** **104**, L140407 (2021).
 - [16] B. Jiang, L. Wang, R. Bi, J. Fan, J. Zhao, D. Yu, Z. Li, and X. Wu, Chirality-dependent hall effect and antisymmetric magnetoresistance in a magnetic weyl semimetal, **Phys. Rev. Lett.** **126**, 236601 (2021).
 - [17] S. Lahiri, T. Bhore, K. Das, and A. Agarwal, Nonlinear magnetoresistivity in two-dimensional systems induced by berry curvature, **Phys. Rev. B** **105**, 045421 (2022).
 - [18] V. Sunko, C. Liu, M. Vila, I. Na, Y. Tang, V. Kozii, S. M. Griffin, J. E. Moore, and J. Orenstein, **Linear magnetoconductivity as a dc probe of time-reversal symmetry breaking** (2023), [arXiv:2310.15631](#).
 - [19] K. Ghorai, S. Das, H. Varshney, and A. Agarwal, Planar hall effect in quasi-two-dimensional materials, **Phys. Rev. Lett.** **134**, 026301 (2025).
 - [20] S. Albarakati, C. Tan, Z.-J. Chen, J. G. Partridge, G. Zheng, L. Farrar, E. L. Mayes, M. R. Field, C. Lee, Y. Wang, *et al.*, Antisymmetric magnetoresistance in van der waals Fe₃GeTe₂/graphite/Fe₃GeTe₂ trilayer heterostructures, **Science advances** **5**, eaaw0409 (2019).
 - [21] Y. Wang, P. A. Lee, D. M. Silevitch, F. Gomez, S. E. Cooper, Y. Ren, J.-Q. Yan, D. Mandrus, T. F. Rosenbaum, and Y. Feng, Antisymmetric linear magnetoresistance and the planar hall effect, **Nature Communications** **11**, 216 (2020).
 - [22] W. Niu, Z. Cao, Y. Wang, Z. Wu, X. Zhang, W. Han, L. Wei, L. Wang, Y. Xu, Y. Zou, L. He, and Y. Pu, Antisymmetric magnetoresistance in Fe₃GeTe₂ nanodevices of inhomogeneous thickness, **Phys. Rev. B** **104**, 125429 (2021).
 - [23] T. C. Fujita, Y. Kozuka, M. Uchida, A. Tsukazaki, T. Arima, and M. Kawasaki, Odd-parity magnetoresistance in pyrochlore iridate thin films with broken time-reversal symmetry, **Scientific Reports** **5**, 9711 (2015).
 - [24] B. Jiang, L. Wang, R. Bi, J. Fan, J. Zhao, D. Yu, Z. Li, and X. Wu, Chirality-dependent hall effect and antisymmetric magnetoresistance in a magnetic weyl semimetal, **Phys. Rev. Lett.** **126**, 236601 (2021).
 - [25] K. Takiguchi, L. D. Anh, T. Chiba, H. Shiratani, R. Fukuzawa, T. Takahashi, and M. Tanaka, Giant gate-controlled odd-parity magnetoresistance in one-dimensional channels with a magnetic proximity effect, **Nature Communications** **13**, 6538 (2022).
 - [26] R. Moubah, F. Magnus, B. Hjörvarsson, and G. Andersson, Antisymmetric magnetoresistance in SmCo₅ amorphous films with imprinted in-plane magnetic anisotropy, **Journal of Applied Physics** **115** (2014).
 - [27] N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald, and N. P. Ong, Anomalous hall effect, **Review of Modern Physics** **82**, 1539 (2010).
 - [28] Nozières, P. and Lewiner, C., A simple theory of the anomalous hall effect in semiconductors, **J. Phys. France** **34**, 901 (1973).
 - [29] M. Liu, L. Ma, G. Li, C. Zhen, D. Hou, and D. Zhao, **Extrinsic suppression of anomalous hall effect in fe-rich kagome magnet fe₃sn** (2024), [arXiv:2411.00746 \[cond-mat.mtrl-sci\]](#).
 - [30] P. Chowdhury, J. Sau, M. Numan, J. Sannigrahi, M. Gutmann, S. Giri, M. Kumar, and S. Majumdar, **Suppression of intrinsic hall effect through competing berry curvature in cr_{1+δ}te₂** (2024), [arXiv:2411.14045 \[cond-mat.mtrl-sci\]](#).
 - [31] K. Das and A. Agarwal, Intrinsic hall conductivities induced by the orbital magnetic moment, **Phys. Rev. B** **103**, 125432 (2021).
 - [32] P. Bhalla, K. Das, D. Culcer, and A. Agarwal, Resonant second-harmonic generation as a probe of quantum geometry, **Phys. Rev. Lett.** **129**, 227401 (2022).
 - [33] P. C. Adak, S. Sinha, A. Agarwal, and M. M. Deshmukh, Tunable moiré materials for probing berry physics and topology, **Nature Reviews Materials** **9**, 481 (2024).
 - [34] F. Pizzocchero, L. Gammelgaard, B. S. Jessen, J. M. Caridad, L. Wang, J. Hone, P. Bøggild, and T. J. Booth, The hot pick-up technique for batch assembly of van der waals heterostructures, **Nature communications** **7**, 11894 (2016).
 - [35] I. A. Verzhbitskiy, H. Kurebayashi, H. Cheng, J. Zhou, S. Khan, Y. P. Feng, and G. Eda, Controlling the magnetic anisotropy in Cr₂Ge₂Te₆ by electrostatic gating, **Nature Electronics** **3**, 460 (2020).
 - [36] C.-C. Tseng, T. Song, Q. Jiang, Z. Lin, C. Wang, J. Suh, K. Watanabe, T. Taniguchi, M. A. McGuire, D. Xiao, *et al.*, Gate-tunable proximity effects in graphene on layered magnetic insulators, **Nano Letters** **22**, 8495 (2022).
 - [37] G. Tencasini, D. Soler-Delgado, Z. Wang, F. Yao, D. Dumcenco, E. Giannini, K. Watanabe, T. Taniguchi, C. Mouldsdales, A. Garcia-Ruiz, *et al.*, Band gap opening in bilayer graphene-CrCl₃/CrBr₃/CrI₃ van der waals interfaces, **Nano letters** **22**, 6760 (2022).
 - [38] Y. Wang, X. Gao, K. Yang, P. Gu, X. Lu, S. Zhang, Y. Gao, N. Ren, B. Dong, Y. Jiang, *et al.*, Quantum hall phase in graphene engineered by interfacial charge coupling, **Nature Nanotechnology** **17**, 1272 (2022).
 - [39] T. K. Chau, S. J. Hong, H. Kang, and D. Suh, Two-dimensional ferromagnetism detected by proximity-coupled quantum hall effect of graphene, **npj Quantum Materials** **7**, 27 (2022).
 - [40] K. Yang, X. Gao, Y. Wang, T. Zhang, Y. Gao, X. Lu, S. Zhang, J. Liu, P. Gu, Z. Luo, *et al.*, Unconventional correlated insulator in CrOCl-interfaced bernal bilayer graphene, **Nature Communications** **14**, 2136 (2023).
 - [41] W. Zhu, V. Perebeinos, M. Freitag, and P. Avouris, Carrier scattering, mobilities, and electrostatic potential in monolayer, bilayer, and trilayer graphene, **Phys. Rev. B** **80**, 235402 (2009).

- [42] T. Khouri, U. Zeitler, C. Reichl, W. Wegscheider, N. Hussey, S. Wiedmann, and J. Maan, Linear magnetoresistance in a quasifree two-dimensional electron gas in an ultrahigh mobility GaAs quantum well, *Phys. Rev. Lett.* **117**, 256601 (2016).
- [43] H. Stormer, K. Baldwin, L. Pfeiffer, and K. West, Strikingly linear magnetic field dependence of the magnetoresistivity in high quality two-dimensional electron systems, *Solid state communications* **84**, 95 (1992).
- [44] S. Iwakiri, L. V. Ginzburg, M. P. Rössli, Y. Meir, A. Stern, C. Reichl, M. Berl, W. Wegscheider, T. Ihn, and K. Ensslin, Nonlinear response of a two-dimensional electron gas in the quantum hall regime, *Phys. Rev. Res.* **5**, L032046 (2023).
- [45] K. Zollner and J. Fabian, Bilayer graphene encapsulated within monolayers of WS₂ or Cr₂Ge₂Te₆: Tunable proximity spin-orbit or exchange coupling, *Phys. Rev. B* **104**, 075126 (2021).
- [46] V. Garcia, Y. Sidis, M. Marangolo, F. Vidal, M. Eddrief, P. Bourges, F. Maccherozzi, F. Ott, G. Panaccione, and V. H. Etgens, Biaxial strain in the hexagonal plane of MnAs thin films: The key to stabilize ferromagnetism to higher temperature, *Phys. Rev. Lett.* **99**, 117205 (2007).
- [47] L. Zhang, X. Huang, H. Dai, M. Wang, H. Cheng, L. Tong, Z. Li, X. Han, X. Wang, L. Ye, and J. Han, Proximity-coupling-induced significant enhancement of coercive field and Curie temperature in 2D van der Waals heterostructures, *Advanced Materials* **32**, 2002032 (2020).
- [48] D. V. Averyanov, I. S. Sokolov, A. M. Tokmachev, O. E. Parfenov, I. A. Karateev, A. N. Taldenkov, and V. G. Storchak, High-temperature magnetism in graphene induced by proximity to EuO, *ACS Applied Materials & Interfaces* **10**, 20767 (2018).
- [49] X.-J. Dong, J.-Y. You, Z. Zhang, B. Gu, and G. Su, Great enhancement of Curie temperature and magnetic anisotropy in two-dimensional van der Waals magnetic semiconductor heterostructures, *Phys. Rev. B* **102**, 144443 (2020).
- [50] Y. Hou, Y. Wei, D. Yang, K. Wang, K. Ren, and G. Zhang, Enhancing the Curie temperature in Cr₂Ge₂Te₆ via charge doping: A first-principles study, *Molecules* **28** (2023).
- [51] Our assumption stems from the physical intuition that the response parameter (α) resulting from TRS breaking should be proportional to the TRS breaking parameter (M). We have also checked this explicitly for the analytically tractable example of gapped graphene. However, concrete and general proof of this statement is still lacking.
- [52] G. T. Lin, H. L. Zhuang, X. Luo, B. J. Liu, F. C. Chen, J. Yan, Y. Sun, J. Zhou, W. J. Lu, P. Tong, Z. G. Sheng, Z. Qu, W. H. Song, X. B. Zhu, and Y. P. Sun, Tricritical behavior of the two-dimensional intrinsically ferromagnetic semiconductor CrGeTe₃, *Phys. Rev. B* **95**, 245212 (2017).
- [53] Y. Liu, Y. Tang, Z. Wang, C. Liu, C. Chen, and J. Wang, Anomalous linear magnetoresistance in high-quality crystalline lead thin films, *Phys. Rev. B* **102**, 140403 (2020).
- [54] A. Aharon-Steinberg, A. Marguerite, D. J. Perello, K. Bagani, T. Holder, Y. Myasoedov, L. S. Levitov, A. K. Geim, and E. Zeldov, Long-range nontopological edge currents in charge-neutral graphene, *Nature* **593**, 528 (2021).
- [55] Y.-T. Yao, S.-Y. Xu, and T.-R. Chang, Atomic scale quantum anomalous Hall effect in monolayer graphene/MnBi₂Te₄ heterostructure, *Materials Horizons* (2024).
- [56] T. Wang, H. Zhang, M. Li, X. Zhao, C. Xia, Y. An, and S. Wei, Strain-controllable high Curie temperature and magnetic anisotropy energy in two-dimensional Fe₂Si and Fe₂Ge, *Physica E: Low-dimensional Systems and Nanostructures* **151**, 115732 (2023).
- [57] M. Rassekh, J. He, S. Farjami Shayesteh, and J. J. Palacios, Remarkably enhanced Curie temperature in monolayer CrI₃ by hydrogen and oxygen adsorption: A first-principles calculations, *Computational Materials Science* **183**, 109820 (2020).
- [58] Y. Wu, Y. Hu, C. Wang, X. Zhou, W. Xia, Y. Zhang, J. Wang, Y. Ding, J. He, P. Dong, S. Bao, J. Wen, Y. Guo, K. Watanabe, T. Taniguchi, W. Ji, Z.-J. Wang, and J. Li, Large enhancement of Curie temperature in Fe₃GeTe₂ by Fe intercalation in the van der Waals gap (2023).
- [59] I. Khan, J. Ahmad, M. E. Mazhar, and J. Hong, Strain tuning of the Curie temperature and valley polarization in two dimensional ferromagnetic WSe₂/Cr₂Se₃ heterostructure, *Nanotechnology* **32**, 375708 (2021).
- [60] J. Zhong, M. Wang, T. Liu, Y. Zhao, X. Xu, S. Zhou, J. Han, L. Gan, and T. Zhai, Strain-sensitive ferromagnetic two-dimensional Cr₂Te₃, *Nano Research* **15**, 12541259 (2021).
- [61] H. Idzuchi, A. E. Llacsahuanga Allcca, A. K. A. Lu, M. Saito, S. Das, J. F. Ribeiro, M. Houssa, R. Meng, K. Inoue, X.-C. Pan, K. Tanigaki, Y. Ikuhara, T. Nakanishi, and Y. P. Chen, Enhanced ferromagnetism in an artificially stretched lattice in quasi-two-dimensional Cr₂Ge₂Te₆, *Phys. Rev. B* **111**, L020402 (2025).
- [62] M. M. Parish and P. B. Littlewood, Non-saturating magnetoresistance in heavily disordered semiconductors, *Nature* **426**, 162165 (2003).
- [63] A. A. Abrikosov, Quantum magnetoresistance, *Phys. Rev. B* **58**, 2788 (1998).
- [64] A. A. Abrikosov, Quantum linear magnetoresistance, *Europhysics Letters (EPL)* **49**, 789793 (2000).
- [65] J. C. W. Song, G. Refael, and P. A. Lee, Linear magnetoresistance in metals: Guiding center diffusion in a smooth random potential, *Phys. Rev. B* **92**, 180204 (2015).
- [66] A. A. Sinchenko, P. D. Grigoriev, P. Lejay, and P. Monceau, Linear magnetoresistance in the charge density wave state of quasi-two-dimensional rare-earth tritellurides, *Phys. Rev. B* **96**, 245129 (2017).
- [67] B. Wu, V. Barrena, F. Mompeán, M. García-Hernández, H. Suderow, and I. Guillamón, Linear nonsaturating magnetoresistance in the nowotny chimney ladder compound Ru₂Sn₃, *Phys. Rev. B* **101**, 205123 (2020).
- [68] Y. Feng, Y. Wang, D. M. Silevitch, J.-Q. Yan, R. Kobayashi, M. Hedo, T. Nakama, Y. Nuki, A. V. Suslov, B. Mihaila, P. B. Littlewood, and T. F. Rosenbaum, Linear magnetoresistance in the low-field limit in density-wave materials, *Proceedings of the National Academy of Sciences* **116**, 11201 (2019).